11. Problem sheet for Set Theory, Winter 2012

Prof. Dr. Peter Koepke, Dr. Philipp Schlicht
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Problem 39.

(a) Suppose $\kappa \in \text{Card}$ and $X \subseteq \kappa$. Show that X is closed in κ if and only if X is closed in the order topology on κ (definition in problem 36).

(b) Let $\kappa \in \text{Card}$, $\text{cof}(\kappa) \geq \omega_1$. Let $(C_{i<\gamma} \mid i < \gamma)$ be a sequence of cub sets C_i in κ and let $\gamma < \text{cof}(\kappa)$. Then $\bigcap_{i<\gamma} C_i$ is cub in κ.

Problem 40. Suppose $\kappa \geq \omega_1$ is regular and $f\colon \kappa \to \kappa$. Show that the set $\{\alpha < \kappa \mid f[\alpha] \subseteq \alpha\}$ is cub in κ. Is every cub subset of κ of this form?

Problem 41. If $X \subseteq \kappa$ is nonstationary, then there exists a regressive function f on X such that $\{\alpha \mid f(\alpha) \leq \gamma\}$ is bounded for every $\gamma < \kappa$.

Problem 42. A train moves from 0 to an uncountable regular cardinal κ. It stops at every ordinal $\alpha < \kappa$. At 0 the train is empty. If there is at least one person on the train at α, then one person leaves the train (we don't know which one). Then α people get on the train. Show with Fodor's Lemma that the train is empty at κ.

There are 6 points for each problem. Please hand in your solutions on Monday, January 7 before the lecture.